ESTIMATION WITH DYNARE

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CEPREMAP

The likelihood of DSGE models

A reduced form state space representation:

$$
\begin{aligned}
y_t^* &= M\bar{y}(\theta) + M\hat{y}_t + N(\theta)x_t + \eta_t \\
\hat{y}_t &= g_y(\theta)\hat{y}_{t-1} + g_u(\theta)u_t \\
E(\eta_t\eta_t') &= V(\theta) \\
E(u_t u_t') &= Q(\theta)
\end{aligned}
$$

The log–likelihood is computed with the Kalman filter.

An augmented state space representation

$$
\begin{aligned}
y_t^* &= M_1\bar{y}(\theta) + M_1\hat{y}_t + M_2 z_t + N(\theta)t + \eta_t \\
\hat{y}_t &= g_y(\theta)\hat{y}_{t-1} + g_u(\theta)u_t \\
z_t &= z_{t-1} + g_{zy}(\theta)\hat{y}_{t-1} + g_{zu}(\theta)u_t \\
E(\eta_t\eta_t') &= V(\theta) \\
E(u_t u_t') &= Q(\theta)
\end{aligned}
$$

Kalman filter

For $t = 1, \ldots, T$

$$\begin{aligned}
v_t &= y_t^* - \bar{y}^* - M\hat{y}_t - Nx_t \\
F_t &= MP_tM' + V \\
K_t &= g_yP_tg_y'F_t^{-1} \\
\hat{y}_{t+1} &= g_y\hat{y}_t + K_tv_t \\
P_{t+1} &= g_yP_t(g_y - K_tM)' + g_uQg_u'
\end{aligned}$$

with y_1 and P_1 given.

$$\ln L\left(\theta|Y_T^*\right) = -\frac{Tk}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}|F_t| - \frac{1}{2}v_t'F_t^{-1}v_t$$

estim_params

Estimated parameters are declared in a `estim_params; ...end;`.
For each estimated parameter, declare the initial value and, optionally, a lower and upper bound.

Example

```
estim_params;
ALPHA1, 0.5, 0, 1;
ALPHA2, 0.5, 0.2, 0.8;
end;
```

varobs and estimation

Observed variables are declared in `varobs`.
Computing the estimation is triggered by `estimation`.
Required option: `datafile`

Example

```
estim_params;
ALPHA1, 0.5, 0, 1;
ALPHA2, 0.5, 0.2, 0.8;
end;

varobs Y, PIE, RS;

estimation(datafile=ddd);
```

Usefull options

first_obs=n : first observation (default: 1)

nobs=n or nobs=([n1:n2]) : number of observations (default: the entire data file)

mode_file : filename of previous results (default: none)

compute_mode : optimization algorithm

- **0** : no optimization
- **1** : Maltab's fmincon
- **2** : Lester Ingber's adaptive simulated annealing
- **3** : Matlab's fminunc
- **4** : Chris Sims' csminwel (default)
- **5** : Marco Ratto's robust optpimizer

mode_check : draws objective function in each parameter direction.

More options

prefilter : 0, no prefiltering; 1, the data are demeaned before estimation (default: 0).

presample : number of initial periods that don't enter into likelihood computation (default: 0).

like_init : initial covariance matrix of state variable prediction. 1, for stationary models, unconditional variance of state variables; 2, for nonstationary models, diffuse prior, diagonal matrix with 10 on the diagonal.

loglinear : computes a log–linear approximation of the model instead of a linear (default) approximation.

More options

optim=() : changes options for Matlab optimizer (see Matlab `optimset` command).

moments_varendo : computes posterior distribution of moments of endogenous variables.

bayesian_irf : computes posterior distribution of IRF's.

smoother : computes posterior distribution of smoothed variables.

filtered_vars : computes posterior distribution of filtered variables.

forcast=n : computes forecasts for n periods.

observation_trends

Linear trends in the observed variables, if they exist, are declared in `observation_trends; ...end;`
For each observation variables, the trend is expressed as a function of model parameters.

Example

```
observation_trends;
Y (gam);
P (mu/gam);
end;
```

Ireland (2004) model

$$
\begin{aligned}
Y_t &= A_t K_t^{\theta} (\eta^t H_t)^{1-\theta} \\
\ln A_t &= (1-\rho) \ln \bar{A} + \rho \ln A_{t-1} + \epsilon_t \\
Y_t &= C_t + I_t \\
K_{t+1} &= (1-\delta) K_t + I_t \\
\gamma C_t H_t &= (1-\theta) Y_t \\
\frac{1}{C_t} &= \beta E_t \left\{ \frac{1}{C_{t+1}} \left[\theta \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta \right] \right\}
\end{aligned}
$$

Stationary variables

$$
\begin{aligned}
\hat{y}_t &= \frac{Y_t}{\eta^t} \\
\hat{c}_t &= \frac{C_t}{\eta^t} \\
\hat{i}_t &= \frac{I_t}{\eta^t} \\
\hat{k}_t &= \frac{K_t}{\eta^t} \\
\hat{h}_t &= H_t \\
\hat{a}_t &= A_t
\end{aligned}
$$

Stationary model

$$
\begin{aligned}
\hat{y}_t &= \hat{a}_t \hat{k}_t^{\theta} \hat{h}_t^{1-\theta} \\
\ln \hat{a}_t &= (1-\rho)\ln \bar{A} + \rho \ln \hat{a}_{t-1} + \epsilon_t \\
\hat{y}_t &= \hat{c}_t + \hat{i}_t \\
\eta \hat{k}_{t+1} &= (1-\delta)\hat{k}_t + \hat{i}_t \\
\gamma \hat{c}_t \hat{h}_t &= (1-\theta)\hat{y}_t \\
\frac{\eta}{\hat{c}_t} &= \beta E_t \left\{ \frac{1}{\hat{c}_{t+1}} \left[\theta \frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} + 1 - \delta \right] \right\}
\end{aligned}
$$

Preparing it for DYNARE

K_t measures the stock of capital at the beginning of period t. We need the stock of capital at the end of the period:

$$\hat{k}_t \longrightarrow \hat{k}_{t-1} = \frac{K_{t-1}}{\eta^t}$$

For log–linearizing, write the model as a function of the log of the variables.

$$y_t = \ln \hat{y}_t, \quad c_t = \ln \hat{c}_t, \quad i_t = \ln \hat{i}_t$$

$$k_t = \ln \hat{k}_t, \quad h_t = \ln \hat{h}_t, \quad a_t = \ln \hat{a}_t$$

Model ready for DYNARE

$$
\begin{aligned}
e^{y_t} &= e^{a_t} e^{k_{t-1}^{\theta}} e^{h_t^{1-\theta}} \\
a_t &= (1-\rho)\ln \bar{A} + \rho a_{t-1} + \epsilon_t \\
e^{y_t} &= e^{c_t} + e^{i_t} \\
\eta e^{k_t} &= (1-\delta) e^{k_{t-1}} + e^{i_t} \\
\gamma e^{c_t} e^{h_t} &= (1-\theta) e^{y_t} \\
\frac{\eta}{e^{c_t}} &= \beta E_t \left\{ \frac{1}{e^{c_{t+1}}} \left[\theta \frac{e^{y_{t+1}}}{e^{k_t}} + 1 - \delta \right] \right\}
\end{aligned}
$$

Measurement equations

$$
\begin{aligned}
\ln Y_t &= y_t + \bar{y} + \ln \eta t + u_{y_t} \\
\ln C_t &= c_t + \bar{c} + \ln \eta t + u_{c_t} \\
\ln H_t &= h_t + \bar{h} + u_{h_t} \\
u_{y_t} &= r_{11} u_{y_{t-1}} + r_{12} u_{c_{t-1}} + r_{13} u_{h_{t-1}} + \epsilon_{y_t} \\
u_{c_t} &= r_{11} u_{y_{t-1}} + r_{12} u_{c_{t-1}} + r_{13} u_{h_{t-1}} + s_{cy} \epsilon_{y_t} + \epsilon_{c_t} \\
u_{h_t} &= r_{11} u_{y_{t-1}} + r_{12} u_{c_{t-1}} + r_{13} u_{h_{t-1}} + s_{hy} \epsilon_{y_t} + s_{hc} \epsilon_{c_t} + \epsilon_{h_t}
\end{aligned}
$$

Priors in DYNARE

NORMAL_PDF	$N(\mu, \sigma)$	R
GAMMA_PDF	$G_2(\mu, \sigma, p_3)$	$[p_3, +\infty)$
BETA_PDF	$B(\mu, \sigma, p_3, p_4)$	$[p_3, p_4]$
INV_GAMMA_PDF	$IG_1(\mu, \sigma)$	R^+
UNIFORM_PDF	$U(p_3, p_4)$	$[p_3, p_4]$

By default, $p_3 = 0$, $p_4 = 1$.

Schorfheide (2000)

- Standard Cash-in-advance model (see also Nason and Cogley, 1994)
- Portfolio adjustment cost model
- Main Results:
  - Standard CIA model outperforms the PAC model in terms of both better in-sample properties and posterior distribution
  - The PAC model outperforms the CIA model when analyzing the response to a money growth shock.

Standard CIA model:

- Three agents: Household, a firm and a financial intermediary
- Decisions are made **after** the current period surprise change in money growth and technology.

The firm

- Technology

$$Y_t = K_t^{\alpha} \left(A_t N_t\right)^{1-\alpha}$$

- Technology shock

$$\ln(A_t) = \gamma + \ln(A_{t-1}) + \varepsilon_{A,t}$$

Maximisation problem

The firm chooses K_{t+1}, labour demand N_t, dividends F_t and loans L_t

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^{t+1} \frac{F_t}{C_{t+1} P_{t+1}} \right]$$

s.t.

$$\begin{aligned} F_t &\leq L_t + P_t \left[K_t^{\alpha} \left(A_t N_t \right)^{1-\alpha} - K_{t+1} + (1-\delta) K_t \right] - W_t N_t - L_t R_{F,t} \\ W_t N_t &\leq L_t \end{aligned}$$

Financial Intermediary

The following maximisation problem:

$$\max_{\{B_t, L_t, D_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^{t+1} \frac{B_t}{C_{t+1} P_{t+1}} \right]$$

s.t.

$$\begin{aligned} B_t &= D_t + R_{F,t} L_t - R_{H,t} D_t - L_t + X_t \\ L_t &\leq X_t + D_t \end{aligned}$$

where B_t is dividends, D_t deposits, $X_t = M_{t+1} - M_t$, and $R_{H,t}$ is the gross deposit interest rate.

Household

The household chooses consumption C_t, hours worked H_t, and (non-negative) deposits D_t so that

$$\max_{\{C_t, H_t, M_{t+1}, D_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[(1-\phi) \ln C_t + \phi \ln(1-H_t) \right] \right]$$

s.t.

$$\begin{aligned} P_t C_t &\leq M_t - D_t + W_t H_t \\ 0 &\leq D_t \\ M_{t+1} &= (M_t - D_t + W_t H_t - P_t C_t) + R_{H,t} D_t + F_t + B_t \end{aligned}$$

Iarket clearing conditions and other shocl

- $m_t = \frac{M_{t+1}}{M_t}$

$$\ln(m_t) = (1-\rho)\ln(m^*) + \rho\ln(m_{t-1}) + \varepsilon_{M,t}$$

- Labour market

$$H_t = N_t$$

- Money market

$$P_t C_t = M_t + X_t$$

- Goods market

$$C_t + (K_{t+1} - (1-\delta)K_t) = K_t^\alpha (A_t H_t)^{1-\alpha}$$

Stationarized model

- All real variables are detrended by the productivity A_t,
- the price level by M_t/A_t
- X_t,and D_t are detrended by M_t.

First-order conditions

- Euler equation

$$
\begin{aligned}
0 \;=\; & E_t \Big\{ -\widehat{P}_t / \left[\widehat{C}_{t+1}\widehat{P}_{t+1} M_t\right] + \beta P_{t+1}\Big[\alpha e^{-\alpha(\gamma+\varepsilon_{t+1})} \\
& \widehat{K}_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + (1-\delta) e^{-\alpha(\gamma+\varepsilon_{t+1})}\Big] / \left[\widehat{c}_{t+2}\widehat{P}_{t+2} m_{t+1}\right] \Big\}
\end{aligned}
$$

- Firm's borrowing constraint

$$\widehat{W}_t = \widehat{L}_t / N_t$$

- Intertemporal labor market optimality condition

$$-\frac{\phi}{1-\phi}\left[\widehat{C}_t \widehat{P}_t / \left(1-N_t\right)\right] + \widehat{L}_t / N_t = 0$$

First order conditions (continued)

- Equilibrium interest rate

$$R_t = (1-\alpha)\widehat{P}_t e^{-\alpha(\gamma+\varepsilon_{t+1})}\widehat{K}_t^{\alpha-1}N_t^{1-\alpha}/\widehat{W}_t$$

- Credit market optimality condition

$$\begin{aligned} 0 \quad = \quad & \left[\widehat{C}_t\widehat{P}_t\right]^{-1} - \beta\left[(1-\alpha)\,\widehat{P}_t e^{-\alpha(\gamma+\varepsilon_{t+1})}\widehat{K}_t^{\alpha-1}N_t^{1-\alpha}/\widehat{L}_t M_t\right] \\ & \times E_t\left[\widehat{C}_{t+1}\widehat{P}_{t+1}\right]^{-1} \end{aligned}$$

- Market clearing conditions . . .

Endogenous variables

$$\begin{bmatrix} \widehat{K}_{t+1} & N_t & \widehat{D}_t & \widehat{C}_t & \widehat{L}_t & \widehat{P}_t \end{bmatrix}$$

Observable variables

One observes output and the price level.

$$\begin{aligned} \ln Y_t - \ln Y_{t-1} &= \ln \widehat{Y}_t - \ln \widehat{Y}_{t-1} + \ln dA_t \\ \ln P_t - \ln P_{t-1} &= \ln \widehat{P}_t - \ln \widehat{P}_{t-1} + \ln m_t - \ln dA_t \end{aligned}$$

One can do either a stationary estimation on the rate of growth of the observed variables or a nonstationary estimation on the level of the observed variables
Differences:

- one more observation
- taking into account cointegration between the observed variables (there is none in this example)

Difficulties with nonstationary models

1. there are either an infinity (pure random walk) or no steady state (random walk with drift).
   One must however compute the steady state of the stationary part of the model.
2. Initialization of the Kalman filter. In the stationary case, the filter is initialized with the unconditional mean and variance of the endogenous variables.
   Use a diffuse Kalman filter as in Durbin and Koopman (2001). This puts a diffuse prior on the initial conditions of the filter.
   In Dynare, the diffuse Kalman filter is used automatically if some variables are declared in the `unit_root_vars` statement.

An augmented state space representation

$$
\begin{aligned}
y_t^* &= M_1\bar{y}(\theta) + M_1\hat{y}_t + M_2 z_t + N(\theta)t + \eta_t \\
\hat{y}_t &= g_y(\theta)\hat{y}_{t-1} + g_u(\theta)u_t \\
z_t &= z_{t-1} + g_{zy}(\theta)\hat{y}_{t-1} + g_{zu}(\theta)u_t \\
E(\eta_t\eta_t') &= V(\theta) \\
E(u_t u_t') &= Q(\theta)
\end{aligned}
$$

Explicit steady state function

The user can supply a Matlab function returning the steady state of the model. This function must be called <*model name*>`_steadystate.m`

- In the returned vector, the variables must be ordered alphabetically.
- It contains steady state value of the stationary variables
- Convenient dummy values for the nonstationary variables (0 for linearization, 1 for loglinearization)

Obtaining posterior distributions

After running Metropolis, Dynare let you obtain posterior distributions for many functions of the parameters. This is controlled by options in the `estimation` statement.

moments_varendo computes posterior distribution of the moments of the endogenous variable as in `stoch_simul` (posterior distribution of the variance decomposition is also included).

bayesian_irf computes and display posterior distribution of IRF's.

smoother computes posterior distribution of the smoothed variables, including the shocks.

forecast=N computes posterior distribution of forecast for N periods after the end of the observation sample. The graph includes one confidence interval describing uncertainty due to parameters and one confidence interval describing uncertainty due to parameters and futur shocks.